STRING SOLUTIONS IN GENERAL BACKGROUNDS

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Motivated by the recent interest in the different aspects of the string/field theory duality, we describe an approach for obtaining exact string solutions in general backgrounds, based on two types of string embedding, allowing for separation of the worldsheet variables τ and σ.

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1 Introduction

Recently, there is a lot of interest in the investigation of the existing connections between different classical string configurations, their semi-classical quantization and the relevant objects in the dual gauge theories, as well as between the corresponding integrable models appearing on the string and field theory sides (see e.g. [1] - [86] [1] and the references therein). In this connection, it seems useful to formulate an approach, which will allow us to obtain exact string solutions in general enough string theory backgrounds. Here, we describe such an approach, based on two types of string embedding, which allow for separation of the worldsheet variables τ and σ.

2 Exact string solutions in general backgrounds

In our further considerations, we will use the Polyakov type action for the bosonic string in a D-dimensional curved space-time with metric tensor $g_{MN}(x)$, interacting with a background 2-form gauge field $b_{MN}(x)$ via Wess-Zumino term

$$S^P = \int d^2\xi \mathcal{L}^P, \quad \mathcal{L}^P = -\frac{1}{2}\left(T\sqrt{-\gamma}\gamma^{mn}G_{mn} - Q\varepsilon^{mn}B_{mn}\right), \tag{2.1}$$
$$\xi^m = (\xi^0, \xi^1) = (\tau, \sigma), \quad m, n = (0, 1),$$

[1] For previously obtained string solutions in curved space-times, see e.g. [88, 89] and the references in [89] and [90].

where

$$G_{mn} = \partial_m X^M \partial_n X^N g_{MN}, \quad B_{mn} = \partial_m X^M \partial_n X^N b_{MN},$$
$$(\partial_m = \partial/\partial\xi^m, \quad M, N = 0, 1, \ldots, D-1),$$

are the fields induced on the string worldsheet, γ is the determinant of the auxiliary worldsheet metric γ_{mn}, and γ^{mn} is its inverse. The position of the string in the background space-time is given by $x^M = X^M(\xi^m)$, and $T = 1/2\pi\alpha'$, Q are the string tension and charge, respectively. If we consider the action (2.1) as a bosonic part of a supersymmetric one, we have to put $Q = \pm T$. In what follows, $Q = T$.

The equations of motion for X^M following from (2.1) are:

$$-g_{LK}\left[\partial_m\left(\sqrt{-\gamma}\gamma^{mn}\partial_n X^K\right) + \sqrt{-\gamma}\gamma^{mn}\Gamma^K_{MN}\partial_m X^M \partial_n X^N\right]$$
$$= \frac{1}{2}H_{LMN}\epsilon^{mn}\partial_m X^M \partial_n X^N, \tag{2.2}$$

where

$$\Gamma_{L,MN} = g_{LK}\Gamma^K_{MN} = \frac{1}{2}\left(\partial_M g_{NL} + \partial_N g_{ML} - \partial_L g_{MN}\right),$$
$$H_{LMN} = \partial_L b_{MN} + \partial_M b_{NL} + \partial_N b_{LM},$$

are the components of the symmetric connection corresponding to the metric g_{MN}, and the field strength of the gauge field b_{MN} respectively. The constraints are obtained by varying the action (2.1) with respect to γ_{mn}:

$$\delta_{\gamma_{mn}} S^P = 0 \Rightarrow \left(\gamma^{kl}\gamma^{mn} - 2\gamma^{km}\gamma^{ln}\right) G_{mn} = 0. \tag{2.3}$$

In solving the equations of motion (2.2) and constraints (2.3), we will use the worldsheet gauge $\gamma_{mn} = constants$. This will allow us to consider the tensionless limit $T \to 0$, corresponding to small t'Hooft coupling $\lambda \to 0$ on the field theory side. Of course, we can always set $\gamma^{mn} = \eta^{mn} = diag(-1, 1)$, to turn to the usually used *conformal gauge*.

We will investigate the string dynamics in the framework of the following two types of embedding:

$$X^\mu(\tau,\sigma) = \Lambda^\mu_0 \tau + \Lambda^\mu_1 \sigma + Y^\mu(\tau), \quad X^a(\tau,\sigma) = Y^a(\tau); \tag{2.4}$$
$$X^\mu(\tau,\sigma) = \Lambda^\mu_0 \tau + \Lambda^\mu_1 \sigma + Z^\mu(\sigma), \quad X^a(\tau,\sigma) = Z^a(\sigma); \tag{2.5}$$
$$\Lambda^\mu_m = const, \quad (m = 0, 1).$$

Here, the embedding coordinates $X^M(\tau,\sigma)$ are divided into $X^M = (X^\mu, X^a)$, where $X^\mu(\tau,\sigma)$ correspond to the space-time coordinates x^μ, on which the background fields do not depend

$$\partial_\mu g_{MN} = 0, \quad \partial_\mu b_{MN} = 0. \tag{2.6}$$

In other words, we suppose that there exist n_μ commuting Killing vectors $\partial/\partial x^\mu$, where n_μ is the number of the coordinates x^μ. In this case, the ansatzes (2.4) and (2.5) allow for separation of the variables τ and σ. By using (2.4), one obtains τ-dependent dynamics, while by using (2.5), one obtains σ-dependent dynamics.

2.1 τ-dependent dynamics

Here, we are going to use the ansatz (2.4) for the string embedding coordinates. In addition, we assume that the conditions (2.6) on the background fields hold.

As far as only two of the constraints (2.3) are independent, we have to choose consistently two of them. Our *independent* constraints, with which we will work in this subsection, are given by

$$\gamma^{00}G_{00} - \gamma^{11}G_{11} = 0 \tag{2.7}$$

and

$$\gamma^{00}G_{01} + \gamma^{01}G_{11} = 0. \tag{2.8}$$

2.1.1 The case $Y^\mu(\tau) = 0$

Let us start with considering the particular case, when in (2.4) $Y^\mu(\tau) = 0$, i.e. X^μ depend on τ and σ linearly. Then, the Lagrangian density, the induced fields, the constraints (2.7) and (2.8) respectively, and the Euler-Lagrange equations for X^M (2.2), can be written as (the over-dot is used for $d/d\tau$)

$$\begin{aligned}\mathcal{L}^A(\tau) = -\frac{T}{2}\sqrt{-\gamma}\Big[&\gamma^{00}g_{ab}\dot{Y}^a\dot{Y}^b + 2\left(\gamma^{0n}g_{a\nu}\Lambda_n^\nu - \frac{1}{\sqrt{-\gamma}}\Lambda_1^\nu b_{a\nu}\right)\dot{Y}^a + \\ &+\gamma^{mn}\Lambda_m^\mu\Lambda_n^\nu g_{\mu\nu} - \frac{2}{\sqrt{-\gamma}}\Lambda_0^\mu\Lambda_1^\nu b_{\mu\nu}\Big];\end{aligned} \tag{2.9}$$

$$\begin{aligned}G_{00} &= g_{ab}\dot{Y}^a\dot{Y}^b + 2\Lambda_0^\nu g_{\nu a}\dot{Y}^a + \Lambda_0^\mu\Lambda_0^\nu g_{\mu\nu}, \\ G_{01} &= \Lambda_1^\nu\left(g_{\nu a}\dot{Y}^a + \Lambda_0^\mu g_{\mu\nu}\right), \quad G_{11} = \Lambda_1^\mu\Lambda_1^\nu g_{\mu\nu};\end{aligned} \tag{2.10}$$

$$B_{01} = -\Lambda_1^\mu\left(b_{\mu a}\dot{Y}^a + \Lambda_0^\nu b_{\mu\nu}\right),$$

$$\gamma^{00}g_{ab}\dot{Y}^a\dot{Y}^b + 2\gamma^{00}\Lambda_0^\nu g_{\nu a}\dot{Y}^a + \left(\gamma^{00}\Lambda_0^\mu\Lambda_0^\nu - \gamma^{11}\Lambda_1^\mu\Lambda_1^\nu\right)g_{\mu\nu} = 0, \tag{2.11}$$

$$\Lambda_1^\nu\left(\gamma^{00}g_{\nu a}\dot{Y}^a + \gamma^{0n}\Lambda_n^\mu g_{\mu\nu}\right) = 0; \tag{2.12}$$

$$\begin{aligned}&\gamma^{00}\left(g_{Lb}\ddot{Y}^b + \Gamma_{L,bc}\dot{Y}^b\dot{Y}^c\right) + 2\gamma^{0n}\Lambda_n^\mu\Gamma_{L,\mu b}\dot{Y}^b + \gamma^{mn}\Lambda_m^\mu\Lambda_n^\nu\Gamma_{L,\mu\nu} \\ &= -\frac{1}{\sqrt{-\gamma}}\Lambda_1^\nu\left(H_{L\mu\nu}\Lambda_0^\mu + H_{La\nu}\dot{Y}^a\right).\end{aligned} \tag{2.13}$$

$\mathcal{L}^A(\tau)$ in (2.9) is like a Lagrangian for a point particle, interacting with the external fields g_{MN}, $b_{a\nu}$ and $b_{\mu\nu}$.

Let us write down the conserved quantities. By definition, the generalized momenta are

$$P_L \equiv \frac{\partial\mathcal{L}}{\partial(\partial_0 X^L)} = -T\left(\sqrt{-\gamma}\gamma^{0n}g_{LN}\partial_n X^N - b_{LN}\partial_1 X^N\right).$$

For our ansatz, they take the form:

$$P_L = -T\left[\sqrt{-\gamma}\left(\gamma^{00}g_{La}\dot{Y}^a + \gamma^{0n}g_{L\nu}\Lambda_n^\nu\right) - b_{L\nu}\Lambda_1^\nu\right].$$

The Lagrangian (2.9) does not depend on the coordinates X^μ. Therefore, the conjugated momenta P_μ are conserved

$$\gamma^{00}g_{\mu a}\dot{Y}^a + \gamma^{0n}\Lambda_n^\nu g_{\mu\nu} - \frac{1}{\sqrt{-\gamma}}\Lambda_1^\nu b_{\mu\nu} = -\frac{P_\mu}{T\sqrt{-\gamma}} = constants. \tag{2.14}$$

The same result can be obtained by solving the equations of motion (2.13) for $L = \lambda$.

From (2.12) and (2.14), one obtains the following compatibility condition

$$\Lambda_1^\nu P_\nu = 0. \tag{2.15}$$

This equality may be interpreted as a solution of the constraint (2.12), which restricts the number of the independent parameters in the theory.

With the help of (2.14), the other constraint, (2.11), can be rewritten in the form

$$g_{ab}\dot{Y}^a\dot{Y}^b = \mathcal{U}, \tag{2.16}$$

where $\mathcal{U}$ is given by

$$\mathcal{U} = \frac{1}{\gamma^{00}}\left[\gamma^{mn}\Lambda_m^\mu\Lambda_n^\nu g_{\mu\nu} + \frac{2\Lambda_0^\mu}{T\sqrt{-\gamma}}\left(P_\mu - T\Lambda_1^\nu b_{\mu\nu}\right)\right]. \tag{2.17}$$

Now, let us turn to the equations of motion (2.13), corresponding to $L = a$. By using the explicit expressions for $\Gamma_{a,\mu b}$, $\Gamma_{a,\mu\nu}$, $H_{a\mu\nu}$ and $H_{ab\nu}$, one obtains

$$g_{ab}\ddot{Y}^b + \Gamma_{a,bc}\dot{Y}^b\dot{Y}^c = \frac{1}{2}\partial_a\mathcal{U} + 2\partial_{[a}\mathcal{A}_{b]}\dot{Y}^b. \tag{2.18}$$

In (2.18), an effective potential $\mathcal{U}$ and an effective gauge field $\mathcal{A}_a$ appeared. $\mathcal{U}$ is given in (2.17), and

$$\mathcal{A}_a = \frac{1}{\gamma^{00}}\left(\gamma^{0m}\Lambda_m^\mu g_{a\mu} - \frac{\Lambda_1^\mu b_{a\mu}}{\sqrt{-\gamma}}\right). \tag{2.19}$$

The reduced equations of motion (2.18) are as for a point particle moving in the gravitational field g_{ab}, in the potential $\mathcal{U}$ and interacting with the 1-form gauge field $\mathcal{A}_a$ through its field strength $\mathcal{F}_{ab} = 2\partial_{[a}\mathcal{A}_{b]}$. The corresponding Lagrangian is

$$\mathcal{L}_{red}^A(\tau) = -\frac{T}{2}\sqrt{-\gamma}\gamma^{00}\left(g_{ab}\dot{Y}^a\dot{Y}^b + 2\mathcal{A}_a\dot{Y}^a + \mathcal{U}\right) + \Lambda_0^\mu P_\mu.$$

Now our task is to find *exact* solutions of the *nonlinear* differential equations (2.16) and (2.18). It turns out that for background fields depending on only one coordinate x^a, we can always integrate these equations, and the solution is [2]

$$\tau\left(X^a\right) = \tau_0 \pm \int_{X_0^a}^{X^a} dx\left(\frac{\mathcal{U}}{g_{aa}}\right)^{-1/2}. \tag{2.20}$$

[2]In this case, the constraint (2.16) is first integral for the equation of motion (2.18).

Otherwise, supposing the metric g_{ab} is a diagonal one, (2.18) and (2.16) reduce to

$$\frac{d}{d\tau}(g_{aa}\dot{Y}^a) - \frac{1}{2}\left[\partial_a g_{aa}(\dot{Y}^a)^2 + \partial_a\mathcal{U}\right] - \frac{1}{2}\sum_{b\neq a}\left[\partial_a g_{bb}(\dot{Y}^b)^2 + 4\partial_{[a}\mathcal{A}_{b]}\dot{Y}^b\right] = 0, \quad (2.21)$$

$$g_{aa}(\dot{Y}^a)^2 + \sum_{b\neq a} g_{bb}(\dot{Y}^b)^2 = \mathcal{U}. \quad (2.22)$$

With the help of the constraint (2.22), we can rewrite the equations of motion (2.21) in the form

$$\frac{d}{d\tau}(g_{aa}\dot{Y}^a)^2 - \dot{Y}^a\partial_a\left(g_{aa}\mathcal{U}\right) + \dot{Y}^a\sum_{b\neq a}\left[\partial_a\left(\frac{g_{aa}}{g_{bb}}\right)(g_{bb}\dot{Y}^b)^2 - 4g_{aa}\partial_{[a}\mathcal{A}_{b]}\dot{Y}^b\right] = 0. \quad (2.23)$$

To find solutions of the above equations without choosing particular background, we can fix all coordinates Y^a except one. Then the *exact* string solution of the equations of motion and constraints is given again by the same expression (2.20) for $\tau\left(X^a\right)$.

To find solutions depending on more than one coordinate, we have to impose further conditions on the background fields. Let us first consider the simpler case, when the last two terms in (2.23) are not present. This may happen, when

$$\partial_a\left(\frac{g_{aa}}{g_{bb}}\right) = 0, \qquad \mathcal{A}_a = 0. \quad (2.24)$$

Then, the first integrals of (2.23) are

$$\left(g_{aa}\dot{Y}^a\right)^2 = D_a(Y^{b\neq a}) + g_{aa}\mathcal{U}, \quad (2.25)$$

where D_a are arbitrary functions of their arguments. These solutions must be compatible with the constraint (2.22), which leads to the condition

$$\sum_a \frac{D_a}{g_{aa}} = (1-n_a)\mathcal{U},$$

where n_a is the number of the coordinates Y^a. From here, one can express one of the functions D_a through the others. To this end, we split the index a in such a way that Y^r is one of the coordinates Y^a, and Y^α are the others. Then

$$D_r = -g_{rr}\left(n_\alpha\mathcal{U} + \sum_\alpha \frac{D_\alpha}{g_{\alpha\alpha}}\right),$$

and by using this, one rewrites the first integrals (2.25) as

$$\left(g_{rr}\dot{Y}^r\right)^2 = g_{rr}\left[(1-n_\alpha)\mathcal{U} - \sum_\alpha \frac{D_\alpha}{g_{\alpha\alpha}}\right] \geq 0, \quad \left(g_{\alpha\alpha}\dot{Y}^\alpha\right)^2 = D_\alpha(Y^{a\neq\alpha}) + g_{\alpha\alpha}\mathcal{U} \geq 0, \quad (2.26)$$

where n_α is the number of the coordinates Y^α. Thus, the constraint (2.22) is satisfied identically.

Now we turn to the general case, when all terms in the equations of motion (2.23) are present. The aim is to find conditions, which will allow us to reduce the order of the equations of motion by one. An example of such *sufficient* conditions, is given below :

$$\mathcal{A}_a \equiv (\mathcal{A}_r, \mathcal{A}_\alpha) = (\mathcal{A}_r, \partial_\alpha f), \quad \partial_\alpha \left(\frac{g_{\alpha\alpha}}{g_{aa}}\right) = 0,$$
$$\partial_\alpha \left(g_{rr}\dot{Y}^r\right)^2 = 0, \quad \partial_r \left(g_{\alpha\alpha}\dot{Y}^\alpha\right)^2 = 0.$$

By using the restrictions given above, one obtains the following first integrals of the equations (2.23), compatible with the constraint (2.22)

$$\left(g_{rr}\dot{Y}^r\right)^2 = g_{rr}\left[(1-n_\alpha)\mathcal{U} - \sum_\alpha \frac{D_\alpha}{g_{\alpha\alpha}} - 2n_\alpha\left(\mathcal{A}_r - \partial_r f\right)\dot{Y}^r\right] = E_r\left(Y^r\right) \geq 0, \quad (2.27)$$
$$\left(g_{\alpha\alpha}\dot{Y}^\alpha\right)^2 = D_\alpha\left(Y^{a\neq\alpha}\right) + g_{\alpha\alpha}\left[\mathcal{U} + 2\left(\mathcal{A}_r - \partial_r f\right)\dot{Y}^r\right] = E_\alpha\left(Y^\beta\right) \geq 0, \quad (2.28)$$

where D_α, E_α and E_r are arbitrary functions of their arguments.

Further progress is possible, when working with particular background configurations, allowing for separation of the variables in (2.26), or in (2.27) and (2.28).

Our results obtained so far are not applicable to tensionless (null) strings, because the action (2.1) is proportional to the string tension T. The parametrization of γ^{mn}, which is appropriate for considering the zero tension limit $T \to 0$, is the following [91, 92]:

$$\gamma^{00} = -1, \quad \gamma^{01} = \lambda^1, \quad \gamma^{11} = (2\lambda^0 T)^2 - (\lambda^1)^2, \quad \det(\gamma^{mn}) = -(2\lambda^0 T)^2. \quad (2.29)$$

Here λ^n are the Lagrange multipliers, whose equations of motion generate the *independent* constraints. In these notations, the constraints (2.11) and (2.12), the equations of motion (2.13), and the conserved momenta (2.14) take the form

$$g_{ab}\dot{Y}^a\dot{Y}^b + 2\Lambda_0^\nu g_{\nu a}\dot{Y}^a + \left\{\Lambda_0^\mu\Lambda_0^\nu + \left[(2\lambda^0 T)^2 - (\lambda^1)^2\right]\Lambda_1^\mu\Lambda_1^\nu\right\} g_{\mu\nu} = 0,$$
$$\Lambda_1^\nu\left[g_{\nu a}\dot{Y}^a + \left(\Lambda_0^\mu - \lambda^1\Lambda_1^\mu\right) g_{\mu\nu}\right] = 0;$$

$$g_{Lb}\ddot{Y}^b + \Gamma_{L,bc}\dot{Y}^b\dot{Y}^c + 2\left(\Lambda_0^\mu - \lambda^1\Lambda_1^\mu\right)\Gamma_{L,\mu b}\dot{Y}^b$$
$$+\left[\left(\Lambda_0^\mu - \lambda^1\Lambda_1^\mu\right)\left(\Lambda_0^\nu - \lambda^1\Lambda_1^\nu\right) - (2\lambda^0 T)^2\Lambda_1^\mu\Lambda_1^\nu\right]\Gamma_{L,\mu\nu} = 2\lambda^0 T\Lambda_1^\nu\left(H_{L\nu b}\dot{Y}^b + \Lambda_0^\mu H_{L\mu\nu}\right);$$

$$g_{\mu a}\dot{Y}^a + \left(\Lambda_0^\nu - \lambda^1\Lambda_1^\nu\right) g_{\mu\nu} + 2\lambda^0 T\Lambda_1^\nu b_{\mu\nu} = 2\lambda^0 P_\mu.$$

The reduced equations of motion and constraint (2.18) and (2.16) have the same form, but now, the effective potential (2.17) and the effective gauge field (2.19) are given by

$$\mathcal{U}^\lambda = \left[\left(\Lambda_0^\mu - \lambda^1\Lambda_1^\mu\right)\left(\Lambda_0^\nu - \lambda^1\Lambda_1^\nu\right) - (2\lambda^0 T)^2\Lambda_1^\mu\Lambda_1^\nu\right] g_{\mu\nu} - 4\lambda^0\Lambda_0^\mu\left(P_\mu - T\Lambda_1^\nu b_{\mu\nu}\right),$$
$$\mathcal{A}_a^\lambda = \left(\Lambda_0^\mu - \lambda^1\Lambda_1^\mu\right) g_{a\mu} + 2\lambda^0 T\Lambda_1^\mu b_{a\mu}.$$

If one sets $\lambda^1 = 0$ and $2\lambda^0 T = 1$, the results in *conformal gauge* are obtained, as it should be. If one puts $T = 0$ in the above formulas, they will describe *tensionless* strings.

2.1.2 The case $Y^\mu(\tau) \neq 0$

By using the ansatz (2.4), one obtains that the Lagrangian density, the induced fields, the constraints (2.7) and (2.8) respectively, and the Euler-Lagrange equations for X^M (2.2) are given by

$$\mathcal{L}^{GA}(\tau) = -\frac{T}{2}\sqrt{-\gamma}\Bigg[\gamma^{00} g_{MN}\dot{Y}^M\dot{Y}^N + 2\left(\gamma^{0n}\Lambda^\nu_n g_{M\nu} - \frac{\Lambda^\nu_1 b_{M\nu}}{\sqrt{-\gamma}}\right)\dot{Y}^M + \\ + \gamma^{mn}\Lambda^\mu_m\Lambda^\nu_n g_{\mu\nu} - \frac{2\Lambda^\mu_0\Lambda^\nu_1 b_{\mu\nu}}{\sqrt{-\gamma}}\Bigg];$$

$$G_{00} = g_{MN}\dot{Y}^M\dot{Y}^N + 2\Lambda^\nu_0 g_{\nu N}\dot{Y}^N + \Lambda^\mu_0\Lambda^\nu_0 g_{\mu\nu}, \\ G_{01} = \Lambda^\nu_1\left(g_{\nu N}\dot{Y}^N + \Lambda^\mu_0 g_{\mu\nu}\right), \quad G_{11} = \Lambda^\mu_1\Lambda^\nu_1 g_{\mu\nu}; \tag{2.30}$$

$$B_{01} = -\Lambda^\mu_1\left(b_{\mu N}\dot{Y}^N + \Lambda^\nu_0 b_{\mu\nu}\right),$$

$$\gamma^{00} g_{MN}\dot{Y}^M\dot{Y}^N + 2\gamma^{00}\Lambda^\nu_0 g_{\nu N}\dot{Y}^N + \left(\gamma^{00}\Lambda^\mu_0\Lambda^\nu_0 - \gamma^{11}\Lambda^\mu_1\Lambda^\nu_1\right) g_{\mu\nu} = 0, \tag{2.31}$$

$$\Lambda^\nu_1\left(\gamma^{00} g_{\nu N}\dot{Y}^N + \gamma^{0n}\Lambda^\mu_n g_{\mu\nu}\right) = 0; \tag{2.32}$$

$$\gamma^{00}\left(g_{LN}\ddot{Y}^N + \Gamma_{L,MN}\dot{Y}^M\dot{Y}^N\right) + 2\gamma^{0n}\Lambda^\mu_n\Gamma_{L,\mu N}\dot{Y}^N + \gamma^{mn}\Lambda^\mu_m\Lambda^\nu_n\Gamma_{L,\mu\nu} = \\ = -\frac{1}{\sqrt{-\gamma}}\Lambda^\nu_1\left(H_{LM\nu}\dot{Y}^M + \Lambda^\mu_0 H_{L\mu\nu}\right). \tag{2.33}$$

The conserved momenta P_μ can be found as before, and now they are

$$\gamma^{00} g_{\mu N}\dot{Y}^N + \gamma^{0n}\Lambda^\nu_n g_{\mu\nu} - \frac{\Lambda^\nu_1 b_{\mu\nu}}{\sqrt{-\gamma}} = -\frac{P_\mu}{T\sqrt{-\gamma}} = constants. \tag{2.34}$$

The compatibility condition following from the constraint (2.32) and from (2.34) coincides with the previous one (2.15). With the help of (2.34), the equations of motion (2.33) corresponding to $L = a$ and the other constraint (2.31), can be rewritten in the form

$$g_{aN}\ddot{Y}^N + \Gamma_{a,MN}\dot{Y}^M\dot{Y}^N = \frac{1}{2}\partial_a\mathcal{U} + 2\partial_{[a}\mathcal{A}_{N]}\dot{Y}^N, \tag{2.35}$$

$$g_{MN}\dot{Y}^M\dot{Y}^N = \mathcal{U}, \tag{2.36}$$

where $\mathcal{U}$ is given by (2.17) and

$$\mathcal{A}_N = \frac{1}{\gamma^{00}}\left(\gamma^{0m}\Lambda^\mu_m g_{N\mu} - \frac{\Lambda^\mu_1 b_{N\mu}}{\sqrt{-\gamma}}\right) \tag{2.37}$$

coincides with (2.19) for $N = a$.

Now we are going to eliminate the variables $\dot{Y}^\mu$ from (2.35) and (2.36). To this end, we express $\dot{Y}^\mu$ through $\dot{Y}^a$ from the conservation laws (2.34):

$$\dot{Y}^\mu = -\frac{\gamma^{0n}}{\gamma^{00}}\Lambda_n^\mu - \left(g^{-1}\right)^{\mu\nu}\left[g_{\nu a}\dot{Y}^a + \frac{1}{\gamma^{00}T\sqrt{-\gamma}}\left(P_\nu - T\Lambda_1^\rho b_{\nu\rho}\right)\right]. \tag{2.38}$$

After using (2.38) and (2.15), the equations of motion (2.35) and the constraint (2.36) acquire the form

$$h_{ab}\ddot{Y}^b + \Gamma^{\mathbf{h}}_{a,bc}\dot{Y}^b\dot{Y}^c = \frac{1}{2}\partial_a\mathcal{U}^{\mathbf{h}} + 2\partial_{[a}\mathcal{A}^{\mathbf{h}}_{b]}\dot{Y}^b, \tag{2.39}$$

$$h_{ab}\dot{Y}^a\dot{Y}^b = \mathcal{U}^{\mathbf{h}}, \tag{2.40}$$

where a new, effective metric appeared

$$h_{ab} = g_{ab} - g_{a\mu}(g^{-1})^{\mu\nu}g_{\nu b}.$$

$\Gamma^{\mathbf{h}}_{a,bc}$ is the symmetric connection corresponding to this metric

$$\Gamma^{\mathbf{h}}_{a,bc} = \frac{1}{2}\left(\partial_b h_{ca} + \partial_c h_{ba} - \partial_a h_{bc}\right).$$

The new effective scalar and gauge potentials, expressed through the background fields, are as follows

$$\mathcal{U}^{\mathbf{h}} = \frac{1}{\gamma\left(\gamma^{00}\right)^2}\left[\Lambda_1^\mu\Lambda_1^\nu g_{\mu\nu} + \frac{1}{T^2}\left(P_\mu - T\Lambda_1^\rho b_{\mu\rho}\right)(g^{-1})^{\mu\nu}\left(P_\nu - T\Lambda_1^\lambda b_{\nu\lambda}\right)\right],$$

$$\mathcal{A}^{\mathbf{h}}_a = -\frac{1}{\gamma^{00}T\sqrt{-\gamma}}\left[g_{a\mu}(g^{-1})^{\mu\nu}\left(P_\nu - T\Lambda_1^\rho b_{\nu\rho}\right) + T\Lambda_1^\rho b_{a\rho}\right].$$

We point out the qualitatively different behavior of the potentials $\mathcal{U}^{\mathbf{h}}$ and $\mathcal{A}^{\mathbf{h}}_a$, compared to $\mathcal{U}$ and $\mathcal{A}_a$, due to the appearance of the inverse metric $(g^{-1})^{\mu\nu}$. The corresponding Lagrangian is

$$\mathcal{L}^{GA}_{red}(\tau) = -\frac{T}{2}\sqrt{-\gamma}\gamma^{00}\left(h_{ab}\dot{Y}^a\dot{Y}^b + 2\mathcal{A}^{\mathbf{h}}_a\dot{Y}^a + \mathcal{U}^{\mathbf{h}}\right) + \frac{d}{d\tau}P_\mu\left(Y^\mu + \Lambda_0^\mu\tau\right).$$

Since the equations (2.18), (2.16) and (2.39), (2.40) have the same form, for obtaining exact string solutions, we can proceed as before and use the previously derived formulas after the replacements $(g, \Gamma, \mathcal{U}, \mathcal{A}) \to (h, \Gamma^{\mathbf{h}}, \mathcal{U}^{\mathbf{h}}, \mathcal{A}^{\mathbf{h}})$. In particular, the solution depending on one of the coordinates X^a will be

$$\tau\left(X^a\right) = \tau_0 \pm \int_{X_0^a}^{X^a} dx\left(\frac{\mathcal{U}^{\mathbf{h}}}{h_{aa}}\right)^{-1/2}. \tag{2.41}$$

In this case by integrating (2.38), and replacing the solution for Y^μ in the ansatz (2.4), one obtains the solution for the string coordinates X^μ:

$$X^\mu(X^a,\sigma) = X_0^\mu + \Lambda_1^\mu\left[\sigma - \frac{\gamma^{01}}{\gamma^{00}}\tau\left(X^a\right)\right] - \tag{2.42}$$
$$- \int_{X_0^a}^{X^a}(g^{-1})^{\mu\nu}\left[g_{\nu a} \pm \frac{\left(P_\nu - T\Lambda_1^\rho b_{\nu\rho}\right)}{\gamma^{00}T\sqrt{-\gamma}}\left(\frac{\mathcal{U}^{\mathrm{h}}}{h_{aa}}\right)^{-1/2}\right]dx.$$

To be able to take the tensionless limit $T \to 0$ in the above formulas, we have to use the λ-parametrization (2.29) of γ^{mn}. The quantities that appear in the reduced equations of motion and constraint (2.39) and (2.40), which depend on this parametrization, are $\mathcal{U}^{\mathbf{h}}$ and $\mathcal{A}_a^{\mathbf{h}}$. Now, they are given by

$$\mathcal{U}^{h,\lambda} = -(2\lambda^0)^2 \left[T^2 \Lambda_1^\mu \Lambda_1^\nu g_{\mu\nu} + (P_\mu - T\Lambda_1^\rho b_{\mu\rho}) (g^{-1})^{\mu\nu} \left(P_\nu - T\Lambda_1^\lambda b_{\nu\lambda}\right)\right],$$
$$\mathcal{A}_a^{h,\lambda} = 2\lambda^0 \left[g_{a\mu}(g^{-1})^{\mu\lambda}(P_\lambda - T\Lambda_1^\rho b_{\lambda\rho}) + T\Lambda_1^\rho b_{a\rho}\right].$$

If one sets $\lambda^1 = 0$ and $2\lambda^0 T = 1$, the *conformal gauge* results are obtained. If one puts $T = 0$ in the above equalities, they will correspond to *tensionless* strings.

2.2 σ-dependent dynamics

In this subsection, we will use the ansatz (2.5) for the string embedding coordinates. Of course, the conditions (2.6) on the background fields are also fulfilled.

Our *independent* constraints, with which we will work in this subsection, are given by

$$\gamma^{00} G_{00} - \gamma^{11} G_{11} = 0, \tag{2.43}$$

and

$$\gamma^{01} G_{00} + \gamma^{11} G_{01} = 0. \tag{2.44}$$

2.2.1 The case $Z^\mu(\sigma) = 0$

Taking into account the conditions (2.6), one obtains the following reduced Lagrangian density, arising from the action (2.1) (the prime is used for $d/d\sigma$)

$$\begin{aligned}\mathcal{L}^A(\sigma) = -\frac{T}{2}\sqrt{-\gamma}\Bigg[\gamma^{11} g_{ab} Z'^a Z'^b + 2\left(\gamma^{1m}\Lambda_m^\mu g_{\mu a} - \frac{1}{\sqrt{-\gamma}}\Lambda_0^\mu b_{\mu a}\right) Z'^a + \\ + \gamma^{mn}\Lambda_m^\mu \Lambda_n^\nu g_{\mu\nu} - \frac{2}{\sqrt{-\gamma}}\Lambda_0^\mu \Lambda_1^\nu b_{\mu\nu}\Bigg],\end{aligned} \tag{2.45}$$

where the fields induced on the string worldsheet are given by

$$\begin{aligned} G_{00} &= \Lambda_0^\mu \Lambda_0^\nu g_{\mu\nu}, \quad G_{01} = \Lambda_0^\mu \left(g_{\mu a} Z'^a + \Lambda_1^\nu g_{\mu\nu}\right), \\ G_{11} &= g_{ab} Z'^a Z'^b + 2\Lambda_1^\mu g_{\mu a} Z'^a + \Lambda_1^\mu \Lambda_1^\nu g_{\mu\nu}; \end{aligned} \tag{2.46}$$

$$B_{01} = \Lambda_0^\mu \left(b_{\mu a} Z'^a + \Lambda_1^\nu b_{\mu\nu}\right);$$

The constraints (2.43) and (2.44) respectively, and the equations of motion for X^M (2.2), can be written as

$$\gamma^{11}\left(g_{ab} Z'^a Z'^b + 2\Lambda_1^\mu g_{\mu b} Z'^b\right) - \left(\gamma^{00}\Lambda_0^\mu \Lambda_0^\nu - \gamma^{11}\Lambda_1^\mu \Lambda_1^\nu\right) g_{\mu\nu} = 0, \tag{2.47}$$

$$\Lambda_0^\mu \left(\gamma^{11} g_{\mu a} Z'^a + \gamma^{1n}\Lambda_n^\nu g_{\mu\nu}\right) = 0; \tag{2.48}$$

$$\gamma^{11}\left(g_{Lb}Z''^b + \Gamma_{L,bc}Z'^bZ'^c\right) + 2\gamma^{1m}\Lambda^\mu_m\Gamma_{L,\mu b}Z'^b + \gamma^{mn}\Lambda^\mu_m\Lambda^\nu_n\Gamma_{L,\mu\nu} \tag{2.49}$$
$$= -\frac{1}{\sqrt{-\gamma}}\Lambda^\mu_0\left(H_{L\mu a}Z'^a + \Lambda^\nu_1 H_{L\mu\nu}\right).$$

Let us write down the conserved quantities. By definition, the generalized momenta are

$$P_L \equiv \frac{\partial\mathcal{L}^P}{\partial(\partial_0 X^L)} = -T\left(\sqrt{-\gamma}\gamma^{0n}g_{LN}\partial_n X^N - b_{LN}\partial_1 X^N\right).$$

For our case, they take the form:

$$P_L = -T\sqrt{-\gamma}\left[\left(\gamma^{01}g_{Lb} - \frac{1}{\sqrt{-\gamma}}b_{Lb}\right)Z'^b + \gamma^{0n}\Lambda^\nu_n g_{L\nu} - \frac{1}{\sqrt{-\gamma}}\Lambda^\nu_1 b_{L\nu}\right].$$

The Lagrangian (2.45) does not depend on the coordinates X^μ. Therefore, the conjugated momenta P_μ do not depend on the proper time τ [3]

$$P_\mu(\sigma) = -T\sqrt{-\gamma}\left[\left(\gamma^{01}g_{\mu b} - \frac{1}{\sqrt{-\gamma}}b_{\mu b}\right)Z'^b + \gamma^{0n}\Lambda^\nu_n g_{\mu\nu} - \frac{1}{\sqrt{-\gamma}}\Lambda^\nu_1 b_{\mu\nu}\right],\ \partial_0 P_\mu = 0. \tag{2.50}$$

In order for our ansatz to be consistent with the action (2.1), the following conditions must be fulfilled

$$\partial_1\mathcal{P}_\mu \equiv \frac{\partial\mathcal{P}_\mu}{\partial\sigma} = 0, \tag{2.51}$$

where

$$\mathcal{P}_M \equiv \frac{\partial\mathcal{L}^P}{\partial(\partial_1 X^M)} = -T\left(\sqrt{-\gamma}\gamma^{1n}g_{MN}\partial_n X^N + b_{MN}\partial_0 X^N\right)$$
$$= -T\sqrt{-\gamma}\left[\gamma^{11}g_{Mb}Z'^b + \gamma^{1n}\Lambda^\nu_n g_{M\nu} + \frac{1}{\sqrt{-\gamma}}\Lambda^\nu_0 b_{M\nu}\right]. \tag{2.52}$$

This is because the equations of motion (2.2) can be rewritten as

$$\frac{\partial P_M}{\partial\tau} + \frac{\partial\mathcal{P}_M}{\partial\sigma} - \frac{\partial\mathcal{L}^P}{\partial x^M} = 0.$$

Hence, for $M = \mu$, these equations take the form (2.51). Therefore, $\mathcal{P}_\mu$ are constants of the motion

$$\gamma^{11}g_{\mu a}Z'^a + \gamma^{1n}\Lambda^\nu_n g_{\mu\nu} + \frac{1}{\sqrt{-\gamma}}\Lambda^\nu_0 b_{\mu\nu} = -\frac{\mathcal{P}_\mu}{T\sqrt{-\gamma}} = constants. \tag{2.53}$$

From (2.48) and (2.53), one obtains the following compatibility condition

$$\Lambda^\nu_0\mathcal{P}_\nu = 0. \tag{2.54}$$

[3] Actually, all momenta P_M do not depend on τ, because there is no such dependence in (2.45).

This equality may be interpreted as a solution of the constraint (2.48), which restricts the number of the independent parameters in the theory.

With the help of (2.53), the other constraint, (2.47), can be rewritten in the form

$$g_{ab}Z'^a Z'^b = \mathcal{U}^s, \tag{2.55}$$

where $\mathcal{U}^s$ is given by

$$\mathcal{U}^s = \frac{1}{\gamma^{11}}\left[\gamma^{mn}\Lambda^\mu_m\Lambda^\nu_n g_{\mu\nu} + \frac{2\Lambda^\mu_1}{T\sqrt{-\gamma}}\left(\mathcal{P}_\mu + T\Lambda^\nu_0 b_{\mu\nu}\right)\right]. \tag{2.56}$$

Now, let us turn to the equations of motion (2.13), corresponding to $L = a$. In view of the conditions (2.6),

$$\Gamma_{a,\mu b} = -\frac{1}{2}\left(\partial_a g_{b\mu} - \partial_b g_{a\mu}\right) = -\partial_{[a}g_{b]\mu}, \quad \Gamma_{a,\mu\nu} = -\frac{1}{2}\partial_a g_{\mu\nu},$$
$$H_{a\mu\nu} = \partial_a b_{\mu\nu}; \quad H_{ab\nu} = \partial_a b_{b\nu} - \partial_b b_{a\nu} = 2\partial_{[a}b_{b]\nu}.$$

By using this, one obtains

$$g_{ab}Z''^b + \Gamma_{a,bc}Z'^b Z'^c = \frac{1}{2}\partial_a\mathcal{U}^s + 2\partial_{[a}\mathcal{A}^s_{b]}Z'^b. \tag{2.57}$$

In (2.57), an effective scalar potential $\mathcal{U}^s$ and an effective 1-form gauge field $\mathcal{A}^s_a$ appeared. $\mathcal{U}^s$ is given in (2.56) (and is the same as in the effective constraint (2.55)), and

$$\mathcal{A}^s_a = \frac{1}{\gamma^{11}}\left(\gamma^{1m}\Lambda^\mu_m g_{a\mu} + \frac{1}{\sqrt{-\gamma}}\Lambda^\mu_0 b_{a\mu}\right). \tag{2.58}$$

The corresponding Lagrangian is

$$\mathcal{L}^A_{red}(\sigma) = -\frac{T}{2}\sqrt{-\gamma}\gamma^{11}\left(g_{ab}Z'^a Z'^b + 2\mathcal{A}^s_a Z'^a + \mathcal{U}^s\right) + \Lambda^\mu_1\mathcal{P}_\mu.$$

Now our task is to find *exact* solutions of the *nonlinear* differential equations (2.55) and (2.57).

If the background seen by the string depends on only one coordinate x^a, the general solution for the string embedding coordinate $X^a(\tau,\sigma) = Z^a(\sigma)$ is given by

$$\sigma\left(X^a\right) = \sigma_0 + \int_{X^a_0}^{X^a}\left(\frac{\mathcal{U}^s}{g_{aa}}\right)^{-1/2} dx.$$

When the background felt by the string depends on more than one coordinate x^a, the first integrals of the equations of motion for $Z^a(\sigma) = (Z^r, Z^\alpha)$, which also solve the constraint (2.55), are

$$\left(g_{rr}Z'^r\right)^2 = g_{rr}\left[(1-n_\alpha)\mathcal{U}^s - 2n_\alpha\left(\mathcal{A}^s_r - \partial_r f\right)Z'^r - \sum_\alpha \frac{D_\alpha\left(Z^{a\neq\alpha}\right)}{g_{\alpha\alpha}}\right] = F_r\left(Z^r\right) \geq 0,$$
$$\left(g_{\alpha\alpha}Z'^\alpha\right)^2 = D_\alpha\left(Z^{a\neq\alpha}\right) + g_{\alpha\alpha}\left[\mathcal{U}^s + 2\left(\mathcal{A}^s_r - \partial_r f\right)Z'^r\right] = F_\alpha\left(Z^\beta\right) \geq 0,$$

where Z^r is one of the coordinates Z^a, Z^α are the remaining ones, n_α is the number of Z^α, and D_α, F_a are arbitrary functions of their arguments. The above expressions are valid, if the g_{ab} part of the metric is diagonal one, and the following integrability conditions hold

$$\mathcal{A}^s_a \equiv (\mathcal{A}^s_r, \mathcal{A}^s_\alpha) = (\mathcal{A}^s_r, \partial_\alpha f), \quad \partial_\alpha \left(\frac{g_{\alpha\alpha}}{g_{aa}}\right) = 0,$$
$$\partial_\alpha \left(g_{rr} Z'^r\right)^2 = 0, \quad \partial_r \left(g_{\alpha\alpha} Z'^\alpha\right)^2 = 0.$$

In the parametrization (2.29) of γ^{mn}, the action (2.1) becomes

$$S_\lambda = \int d^2\xi \Big\{ \frac{1}{4\lambda^0}\Big[G_{00} - 2\lambda^1 G_{01} + \left(\lambda^1\right)^2 G_{11} - \left(2\lambda^0 T\right)^2 G_{11}\Big] + T B_{01}\Big\}.$$

In these notations, the constraints (2.47) and (2.48), the equations of motion (2.49), and the conserved quantities (2.50), (2.53) take the form

$$g_{ab} Z'^a Z'^b + 2\Lambda^\mu_1 g_{\mu b} Z'^b + \left[\frac{\Lambda^\mu_0 \Lambda^\nu_0}{(2\lambda^0 T)^2 - (\lambda^1)^2} + \Lambda^\mu_1 \Lambda^\nu_1\right] g_{\mu\nu} = 0,$$
$$\Lambda^\mu_0 \left\{ g_{\mu a} Z'^a + \left[\frac{\lambda^1 \Lambda^\nu_0}{(2\lambda^0 T)^2 - (\lambda^1)^2} + \Lambda^\nu_1\right] g_{\mu\nu} \right\} = 0;$$

$$g_{Lb} Z''^b + \Gamma_{L,bc} Z'^b Z'^c + 2\left[\frac{\lambda^1 \Lambda^\mu_0}{(2\lambda^0 T)^2 - (\lambda^1)^2} + \Lambda^\mu_1\right] \Gamma_{L,\mu b} Z'^b$$
$$+ \left[\frac{\Lambda^\mu_0 \left(2\lambda^1 \Lambda^\nu_1 - \Lambda^\nu_0\right)}{(2\lambda^0 T)^2 - (\lambda^1)^2} + \Lambda^\mu_1 \Lambda^\nu_1\right] \Gamma_{L,\mu\nu} = -\frac{2\lambda^0 T}{(2\lambda^0 T)^2 - (\lambda^1)^2} \Lambda^\mu_0 \left(H_{L\mu a} Z'^a + \Lambda^\nu_1 H_{L\mu\nu}\right).$$

$$P_\mu(\sigma) = \frac{1}{2\lambda^0}\left[\left(-\lambda^1 g_{\mu a} + 2\lambda^0 T b_{\mu a}\right) Z'^a + \left(\Lambda^\nu_0 - \lambda^1 \Lambda^\nu_1\right) g_{\mu\nu} + 2\lambda^0 T \Lambda^\nu_1 b_{\mu\nu}\right],$$
$$\mathcal{P}_\mu = -\frac{1}{2\lambda^0}\left\{\left[(2\lambda^0 T)^2 - (\lambda^1)^2\right]\left(g_{\mu a} Z'^a + \Lambda^\nu_1 g_{\mu\nu}\right) + \Lambda^\nu_0 \left(\lambda^1 g_{\mu\nu} + 2\lambda^0 T b_{\mu\nu}\right)\right\}.$$

The reduced equations of motion and constraint (2.57) and (2.55) have the same form, but now, the effective potential (2.56) and the effective gauge field (2.58) are given by

$$\mathcal{U}^{s,\lambda} = \left[\frac{\Lambda^\mu_0 \left(2\lambda^1 \Lambda^\nu_1 - \Lambda^\nu_0\right)}{(2\lambda^0 T)^2 - (\lambda^1)^2} + \Lambda^\mu_1 \Lambda^\nu_1\right] g_{\mu\nu} + \frac{4\lambda^0}{(2\lambda^0 T)^2 - (\lambda^1)^2} \Lambda^\mu_1 \left(\mathcal{P}_\mu + T \Lambda^\nu_0 b_{\mu\nu}\right),$$
$$\mathcal{A}^{s,\lambda}_a = \left[\frac{\lambda^1 \Lambda^\nu_0}{(2\lambda^0 T)^2 - (\lambda^1)^2} + \Lambda^\nu_1\right] g_{a\nu} + \frac{2\lambda^0 T}{(2\lambda^0 T)^2 - (\lambda^1)^2} \Lambda^\mu_0 b_{a\mu}.$$

If one sets $\lambda^1 = 0$ and $2\lambda^0 T = 1$, this will correspond to *conformal gauge*, as it should be. If one puts $T = 0$ in the above formulas, they will describe *tensionless* strings.

2.2.2 The case $Z^\mu(\sigma) \neq 0$

Taking into account the ansatz (2.5), one obtains that the induced fields G_{mn} and B_{mn}, the Lagrangian density, the constraints (2.43) and (2.44) respectively, and the Euler-Lagrange equations for X^M (2.2) are given by

$$
\begin{aligned}
G_{00} &= \Lambda_0^\mu \Lambda_0^\nu g_{\mu\nu}, \quad G_{01} = \Lambda_0^\mu \left(g_{\mu N} Z'^N + \Lambda_1^\nu g_{\mu\nu} \right), \\
G_{11} &= g_{MN} Z'^M Z'^N + 2\Lambda_1^\mu g_{\mu N} Z'^N + \Lambda_1^\mu \Lambda_1^\nu g_{\mu\nu};
\end{aligned}
\tag{2.59}
$$

$$B_{01} = \Lambda_0^\mu \left(b_{\mu N} Z'^N + \Lambda_1^\nu b_{\mu\nu} \right);$$

$$
\begin{aligned}
\mathcal{L}^{GA}(\sigma) = -\frac{T}{2}\sqrt{-\gamma} \Bigg[& \gamma^{11} g_{MN} Z'^M Z'^N + 2 \left(\gamma^{1m} \Lambda_m^\mu g_{\mu N} - \frac{\Lambda_0^\mu b_{\mu N}}{\sqrt{-\gamma}} \right) Z'^N + \\
& + \gamma^{mn} \Lambda_m^\mu \Lambda_n^\nu g_{\mu\nu} - \frac{2\Lambda_0^\mu \Lambda_1^\nu b_{\mu\nu}}{\sqrt{-\gamma}} \Bigg];
\end{aligned}
$$

$$\gamma^{11} g_{MN} Z'^M Z'^N + 2\gamma^{11} \Lambda_1^\mu g_{\mu N} Z'^N - \left(\gamma^{00} \Lambda_0^\mu \Lambda_0^\nu - \gamma^{11} \Lambda_1^\mu \Lambda_1^\nu \right) g_{\mu\nu} = 0, \tag{2.60}$$

$$\Lambda_0^\mu \left(\gamma^{11} g_{\mu N} Z'^N + \gamma^{1n} \Lambda_n^\nu g_{\mu\nu} \right) = 0; \tag{2.61}$$

$$
\begin{aligned}
\gamma^{11} \left(g_{LN} Z''^N + \Gamma_{L,MN} Z'^M Z'^N \right) + 2\gamma^{1m} \Lambda_m^\mu \Gamma_{L,\mu N} Z'^N + \gamma^{mn} \Lambda_m^\mu \Lambda_n^\nu \Gamma_{L,\mu\nu} = \\
= -\frac{1}{\sqrt{-\gamma}} \Lambda_0^\mu \left(H_{L\mu N} Z'^N + \Lambda_1^\nu H_{L\mu\nu} \right).
\end{aligned}
\tag{2.62}
$$

The quantities P_L, $\mathcal{P}_L$ can be found as before, and now they are

$$\left(\gamma^{01} g_{LN} - \frac{b_{LN}}{\sqrt{-\gamma}} \right) Z'^N + \gamma^{0n} \Lambda_n^\nu g_{L\nu} - \frac{\Lambda_1^\nu b_{L\nu}}{\sqrt{-\gamma}} = -\frac{P_L}{T\sqrt{-\gamma}}, \quad \partial_0 P_L = 0, \tag{2.63}$$

$$\gamma^{11} g_{LN} Z'^N + \gamma^{1n} \Lambda_n^\nu g_{L\nu} + \frac{\Lambda_0^\nu b_{L\nu}}{\sqrt{-\gamma}} = -\frac{\mathcal{P}_L}{T\sqrt{-\gamma}}, \quad \partial_0 \mathcal{P}_L = 0, \quad \partial_1 \mathcal{P}_\mu = 0. \tag{2.64}$$

The compatibility condition following from the constraint (2.61) and from (2.64) coincides with the previous one (2.54).

As in the previous subsection, the equations (2.62) for $L = \lambda$ lead to $\partial_1 \mathcal{P}_\lambda = 0$. Consequently, our next task is to consider the equations (2.62) for $L = a$ and the constraint (2.60). First of all, we will eliminate the variables Z'^μ from them. To this end, we express Z'^μ through Z'^a by using (2.64):

$$Z'^\mu = -\frac{\gamma^{1m}}{\gamma^{11}} \Lambda_m^\mu - \left(g^{-1} \right)^{\mu\nu} \left[g_{\nu a} Z'^a + \frac{1}{T\sqrt{-\gamma}\gamma^{11}} \left(\mathcal{P}_\nu + T\Lambda_0^\rho b_{\nu\rho} \right) \right]. \tag{2.65}$$

With the help of (2.65) and (2.54), the equations (2.62) for $L = a$ and the constraint (2.60) acquire the form

$$h_{ab} Z''^b + \Gamma^{\mathbf{h}}_{a,bc} Z'^b Z'^c = \frac{1}{2} \partial_a \mathcal{U}^{\mathbf{g}} + 2\partial_{[a} \mathcal{A}^{\mathbf{g}}_{b]} Z'^b, \tag{2.66}$$

$$h_{ab} Z'^a Z'^b = \mathcal{U}^{\mathbf{g}}. \tag{2.67}$$

The effective scalar and gauge potentials, expressed through the background fields, are as follows

$$\mathcal{U}^{\mathbf{g}} = \frac{1}{\gamma\left(\gamma^{11}\right)^2}\left[\Lambda_0^\mu\Lambda_0^\nu g_{\mu\nu} + \frac{1}{T^2}\left(\mathcal{P}_\mu + T\Lambda_0^\rho b_{\mu\rho}\right)(g^{-1})^{\mu\nu}\left(\mathcal{P}_\nu + T\Lambda_0^\lambda b_{\nu\lambda}\right)\right],$$
$$\mathcal{A}_a^{\mathbf{g}} = -\frac{1}{T\sqrt{-\gamma}\gamma^{11}}\left[g_{a\mu}(g^{-1})^{\mu\nu}\left(\mathcal{P}_\nu + T\Lambda_0^\rho b_{\nu\rho}\right) - T\Lambda_0^\rho b_{a\rho}\right].$$

The corresponding Lagrangian is

$$\mathcal{L}_{red}^{GA}(\sigma) = -\frac{T}{2}\sqrt{-\gamma}\gamma^{11}\left(h_{ab}Z'^a Z'^b + 2\mathcal{A}_a^{\mathbf{g}} Z'^a + \mathcal{U}^{\mathbf{g}}\right) + \frac{d}{d\sigma}\mathcal{P}_\mu\left(Z^\mu + \Lambda_1^\mu\sigma\right).$$

Since the equations (2.57), (2.55) and (2.66), (2.67) have the same form, for obtaining exact string solutions, we can proceed as before and use the derived formulas after the replacements $(g, \Gamma, \mathcal{U}^s, \mathcal{A}^s) \to (h, \Gamma^{\mathbf{h}}, \mathcal{U}^{\mathbf{g}}, \mathcal{A}^{\mathbf{g}})$. In particular, the solution depending on one of the coordinates X^a will be

$$\sigma\left(X^a\right) = \sigma_0 + \int_{X_0^a}^{X^a} dx\left(\frac{\mathcal{U}^{\mathbf{g}}}{h_{aa}}\right)^{-1/2}. \tag{2.68}$$

In this case by integrating (2.65), and replacing the solution for Z^μ in the ansatz (2.5), one obtains solution for the string coordinates X^μ of the type $X^\mu(\tau, X^a)$:

$$X^\mu(\tau, X^a) = X_0^\mu + \Lambda_0^\mu\left[\tau - \frac{\gamma^{01}}{\gamma^{11}}\sigma\left(X^a\right)\right] - \tag{2.69}$$
$$-\int_{X_0^a}^{X^a}(g^{-1})^{\mu\nu}\left[g_{\nu a} + \frac{\left(\mathcal{P}_\nu + T\Lambda_0^\rho b_{\nu\rho}\right)}{T\sqrt{-\gamma}\gamma^{11}}\left(\frac{\mathcal{U}^{\mathbf{g}}}{h_{aa}}\right)^{-1/2}\right] dx.$$

To write down a solution of the type $X^\mu(\tau, \sigma)$, one have to invert the solution (2.68): $\sigma\left(X^a\right) \to X^a(\sigma)$. Then, $X^\mu(\tau, \sigma)$ are given by

$$X^\mu(\tau, \sigma) = X_0^\mu + \Lambda_0^\mu\left(\tau - \frac{\gamma^{01}}{\gamma^{11}}\sigma\right) - \tag{2.70}$$
$$-\int_{\sigma_0}^{\sigma}(g^{-1})^{\mu\nu}\left[\frac{\left(\mathcal{P}_\nu + T\Lambda_0^\rho b_{\nu\rho}\right)}{T\sqrt{-\gamma}\gamma^{11}} + g_{\nu a}\left(\frac{\mathcal{U}^{\mathbf{g}}}{h_{aa}}\right)^{1/2}\right] d\sigma.$$

Let us also give the expression for P_μ after the elimination of Z'^μ from (2.63)

$$P_\mu(\sigma) = T\left[b_{\mu a} - b_{\mu\nu}\left(g^{-1}\right)^{\nu\lambda} g_{\lambda a}\right] Z'^a \tag{2.71}$$
$$+\frac{1}{\gamma^{11}}\left\{\gamma^{01}\mathcal{P}_\mu + \frac{1}{\sqrt{-\gamma}}\left[T\Lambda_0^\nu g_{\mu\nu} - b_{\mu\nu}\left(g^{-1}\right)^{\nu\lambda}\left(\mathcal{P}_\lambda + T\Lambda_0^\rho b_{\lambda\rho}\right)\right]\right\}.$$

These equalities connect the conserved momenta P_μ with the constants of the motion $\mathcal{P}_\mu$.

To be able to take the tensionless limit $T \to 0$ in the above formulas, we must use the λ-parametrization (2.29) of γ^{mn}. The quantities, which depend on this parametrization,

and appear in the reduced equations of motion and constraint (2.66), (2.67), and therefore - in the solutions, are $\mathcal{U}^{\mathbf{g}}$ and $\mathcal{A}_a^{\mathbf{g}}$. Now, they read

$$\mathcal{U}^{\mathbf{g}} = -\frac{(2\lambda^0)^2}{[(2\lambda^0 T)^2 - (\lambda^1)^2]^2}\left[T^2\Lambda_0^\mu\Lambda_0^\nu g_{\mu\nu} + \left(\mathcal{P}_\mu + T\Lambda_0^\rho b_{\mu\rho}\right)(g^{-1})^{\mu\nu}\left(\mathcal{P}_\nu + T\Lambda_0^\lambda b_{\nu\lambda}\right)\right],$$
$$\mathcal{A}_a^{\mathbf{g}} = -\frac{2\lambda^0}{(2\lambda^0 T)^2 - (\lambda^1)^2}\left[g_{a\mu}(g^{-1})^{\mu\nu}\left(\mathcal{P}_\nu + T\Lambda_0^\rho b_{\nu\rho}\right) - T\Lambda_0^\rho b_{a\rho}\right].$$

If one sets $\lambda^1 = 0$ and $2\lambda^0 T = 1$, the *conformal gauge* results are obtained. If one puts $T = 0$ in the above equalities, they will correspond to *tensionless* strings. For instance, the solution $X^\mu(\tau, X^a)$ reduces to

$$X^\mu(\tau, X^a)_{T=0} = X_0^\mu + \Lambda_0^\mu\left[\tau + \frac{\sigma\left(X^a\right)_{T=0}}{\lambda^1}\right] -$$
$$-\int_{X_0^a}^{X^a}(g^{-1})^{\mu\nu}\left[g_{\nu a} - \frac{2\lambda^0}{(\lambda^1)^2}\mathcal{P}_\nu\left(\frac{\mathcal{U}^{\mathbf{g}}}{h_{aa}}\right)_{T=0}^{-1/2}\right]dx.$$

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